

# Stability Analysis of Self-Injection-Locked Oscillators

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**Abstract**—This paper addresses the stability analysis of the self-injection-locked oscillators. The analysis is developed for arbitrary self-injection feedback loops and illustrated with the specific case of a simple time-delay cable. It is shown that the output phase stability in self-injection-locked oscillators depends on the feedback loop delay and the types of oscillator circuits, which are represented by equivalent parallel- or series-resonant oscillator models. The self-injection-locked technique can also be used to test the oscillator circuit model when the self-coupling phase is known. The theory is verified by using a self-injection-locked GaAs MESFET oscillator operating at *X*-band with delay loops.

**Index Terms**—AM noise, delay line, feedback loop, injection lock, noise, oscillator, parallel resonant, phase (PM) noise, resonator, self-injection lock, series resonant, stability.

## I. INTRODUCTION

STABLE microwave and millimeter-wave sources in communication systems are required for commercial and military applications. Similarly, commercial digital communication systems also put strict constraints on the signal-to-noise ratio and bit error rate (BER) for high-fidelity information transmission. Stable oscillators typically use the very high-*Q* external resonator in the oscillator circuits, or pass the oscillator output signal through the high-*Q* cavity to stabilize the signals and eliminate the noise components [1]–[4]. The higher effective *Q* factor of the oscillator means fewer fluctuations for the oscillator output phase and frequency, and the oscillator is more stable. Another way to reduce the phase noise and stabilize the oscillator phase and frequency is to injection lock the oscillator with an external low-noise signal [5]–[7]. The noise reduction in single one-oscillator and coupled-oscillator arrays phase locked to the external low phase-noise signal has been verified in the authors' previous work by theories and experiments [5]–[7].

There is another way to stabilize the oscillators and reduce the noise by using self-injection-locked technique [8]–[10]. A part of the oscillator output signal is used to injection lock the oscillator itself (Fig. 1). The self-injection signal has the same frequency as the oscillator, and it is easy for the oscillator to remain phase locked all the time as long as it satisfies the stability conditions. The oscillator phase fluctuation or noise can be reduced in self-injection-locked oscillators for certain conditions [11], and the equivalent *Q* factor is also increased to stabilize the output signal phase. In the previous literatures, the dependences of the stability of the self-injection-locked oscillator on the oscillator circuit models and feedback delay are not yet addressed.

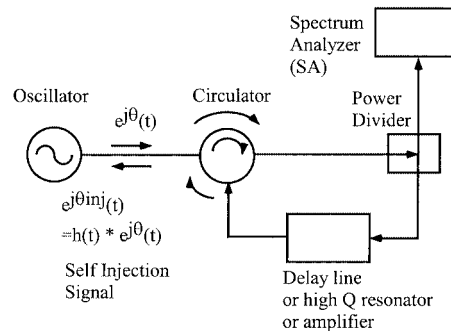


Fig. 1. Setup for the self-injection-locked oscillator. The oscillator output signal goes through the circulator, and then into the input port of the power divider. Two output ports of the power divider are connected to the spectrum analyzer and the output load with feedback loop to the circulator, respectively. A part of the oscillator output undelayed signal is feedback to the circulator as the self-injection (delayed) signal. In the figure,  $h(t)$  is the self-injection-locked feedback transfer function in the time domain. In the feedback loop, the delay cable, high-*Q* factor resonator, or amplifier may be used. The attenuator may be inserted into the loop to change the self-injection signal strength.

In this paper, the author analyzes the stability of the self-injection-locked oscillators in theory and verify the results by experiments. The author extends the previous work on the stable mode analysis of coupled oscillators and explores the stability dependence [12] on the oscillator circuit models and self-injection signal delay in this paper.

## II. PHASE DYNAMICS OF A SINGLE OSCILLATOR WITH A SELF-INJECTION SIGNAL

The oscillator circuits can be modeled and classified as either parallel- or series-resonant oscillators. The parallel-resonant oscillator model, shown in Fig. 2(a), has a negative conductance, an output load, and an *L*–*C* resonator, and the series-resonant oscillator model with negative resistance, an output load, and an *L*–*C* resonator, as shown in Fig. 2(b). The phase dynamics of the self-injection-locked oscillator in the parallel resonant oscillators are different from those in series resonant oscillator models, and also depend on the coupling phase [12]. In the following, the phase dynamics and stability analysis will be derived for parallel and series resonant oscillator models.

### A. Parallel-Resonant Oscillators

If the oscillator is locked to a low-power injection signal, the phase relationship between the parallel-resonant oscillator output signal and injection signal can be described as [1]–[3], [12]

$$\frac{d\theta(t)}{dt} = \omega_0 + \rho \frac{\omega_0}{2Q} \sin(\psi_{inj}(t) - \theta(t) + \Phi) \quad (1)$$

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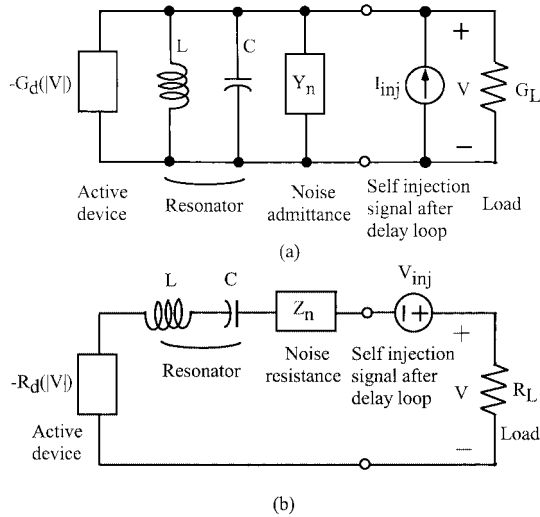


Fig. 2. (a) Equivalent parallel-resonant oscillator model. The model includes an active device, noise admittance, parallel  $L$ - $C$  resonator, output load, and self-injection signal. The noise admittance characterizes the intrinsic noise sources in the parallel-resonant oscillator circuits. (b) Equivalent series-resonant oscillator model. The model includes an active device, noise resistance, series  $L$ - $C$  resonator, output load, and self-injection signal. The noise resistance characterizes the intrinsic noise sources in the series-resonant oscillator circuits.

where  $\theta$  and  $\psi_{inj}$  are the instantaneous phases of the oscillator output signal and injection signals, respectively.  $\omega_0$  and  $Q$  are the free-running frequency and  $Q$  factor of the oscillator, respectively.  $\Phi$  is the coupling phase or delay from the injection signal source and the slave oscillator.  $\rho = A_{inj}/A$  is the injection strength, and the injection signal  $A_{inj}$  is normalized to the oscillator's free-running amplitude  $A$ .

A steady-state noise-free synchronized state for parallel-resonant oscillators satisfies

$$\sin(\hat{\psi}_{inj}(t) - \hat{\theta}(t) + \Phi) = \frac{\omega_{inj} - \omega_0}{\rho\omega_{3dB}} = \frac{\omega_{inj} - \omega_0}{\Delta\omega_{lock}} \quad (2)$$

where  $d\theta/dt = \omega_{inj}$  is the injection frequency,  $\omega_{3dB} = \omega_0/(2Q)$  is half the 3-dB frequency of the oscillator tank circuits,  $\Delta\omega_{lock} = \rho\omega_{3dB}$  is half the entire locking range, and the circumflex (^) denotes a steady-state quantity.

If we extract a part of the oscillator output signal and feed it through a feedback with the transfer function [the frequency response is  $H(\omega)$  and the time-domain response is  $h(t)$ ] shown in Fig. 1 and then into the oscillator injection port, the phase difference in a sinusoidal term can be written as  $\psi_{inj}(t) + \Phi - \theta(t) = \theta_{inj}(t) - \theta(t) = \theta(t) * h(t) - \theta(t) = \theta(t - T) - \theta(t)$ , where  $(*)$  is the convolution symbol and  $T$  is the feedback delay time. Here, the feedback transfer function ( $H(\omega)$  or  $h(t)$ ) only introduces the phase or time delay into the signal phase, but does not change the signal frequency. Therefore, the phase relationship of (1) becomes

$$\frac{d(\Delta\theta_p(t))}{dt} = \left( \omega_0 - \frac{d\theta(t-T)}{dt} \right) - \rho\omega_{3dB} \sin(\Delta\theta_p(t)) \quad (3)$$

where  $\Delta\theta_p(t) = \theta(t) - \theta(t-T)$  and  $\omega_0$  is the carrier frequency. The subscript  $p$  denotes the parallel-resonant oscillator.

Differential equation (3) is an implicit equation of  $\theta(t)$ , and one needs to use the numerical method to find the exact steady state solution with the initial value  $\theta_0$ . However, the locking range  $\rho\omega_{3dB} \ll \omega_0$  in real self-injection-locked oscillators and the steady-state phase can be approximated by  $\theta(t) \approx \omega_0 t + \theta_0$ .

### B. Series-Resonant Oscillators

If the oscillator is locked to a low-power injection signal, the phase relationship between the series-resonant oscillator output signal and injection signal can be described as [12]

$$\frac{d\theta(t)}{dt} = \omega_0 - \rho \frac{\omega_0}{2Q} \sin(\psi_{inj}(t) - \theta(t) + \Phi) \quad (4)$$

where  $\theta$  and  $\psi_{inj}$  are the instantaneous phases of the oscillator output signal and the injection signals, respectively.  $\omega_0$  and  $Q$  are the free-running frequency and  $Q$  factor of the oscillator, respectively.  $\Phi$  is the coupling phase or delay from the injection signal source and the slave oscillator.  $\rho = A_{inj}/A$  is the injection strength and the injection signal  $A_{inj}$  is normalized to the oscillator's free-running amplitude  $A$ .

A steady-state noise-free synchronized state for series-resonant oscillators is

$$\sin(\hat{\psi}_{inj}(t) - \hat{\theta}(t) + \Phi) = -\frac{(\omega_{inj} - \omega_0)}{\rho\omega_{3dB}} = -\frac{(\omega_{inj} - \omega_0)}{\Delta\omega_{lock}} \quad (5)$$

where  $d\theta/dt = \omega_{inj}$  is the injection frequency,  $\omega_{3dB} = \omega_0/(2Q)$  is half the 3-dB frequency of the oscillator tank circuits,  $\Delta\omega_{lock} = \rho\omega_{3dB}$  is half the entire locking range, and the circumflex (^) denotes a steady-state quantity.

If we extract a part of the oscillator output signal and feed it through a feedback loop with the transfer function (the frequency response is  $H(\omega)$  and the time-domain response is  $h(t)$ , shown in Fig. 1) and then into the oscillator injection port, the phase difference in the sinusoidal term can be written as  $\psi_{inj}(t) + \Phi - \theta(t) = \theta_{inj}(t) - \theta(t) = \theta(t) * h(t) - \theta(t) = \theta(t - T) - \theta(t)$ , where  $(*)$  is the convolution symbol and  $T$  is the feedback delay time. Here, the feedback transfer function [ $H(\omega)$  or  $h(t)$ ] only introduces the phase or time delay into the signal phase, but does not change the signal frequency. Therefore, the phase relationship of (4) becomes

$$\frac{d(\Delta\theta_s(t))}{dt} = \left( \omega_0 - \frac{d\theta(t-T)}{dt} \right) + \rho\omega_{3dB} \sin(\Delta\theta_s(t)) \quad (6)$$

where  $\Delta\theta_s(t) = \theta(t) - \theta(t-T)$  and  $\omega_0$  is the carrier frequency. The subscript  $s$  denotes the series-resonant oscillators.

### III. STABILITY ANALYSIS OF THE SELF-INJECTION-LOCKED OSCILLATORS

The stability of the solutions to the phase dynamics can be determined by linearizing the equations around the fixed points. One can assume that the steady-state solutions to (3) and (6) are  $\Delta\hat{\theta}_p$  and  $\Delta\hat{\theta}_s$ , respectively, which are the phase difference (or phase delay) between the oscillator output signal phase and the injection signal phase at the injection port for parallel and series-resonant oscillators, respectively.

The stability analysis to the solution of phase dynamics for self-injection-locked oscillators from (3) or (6) can be written as

$$\frac{d(\Delta\hat{\theta} + \delta)}{dt} = \left( \frac{d\hat{\theta}(t-T)}{dt} - \omega_0 \right) \mp \rho\omega_{3dB} \sin(\Delta\hat{\theta} + \delta) \quad (7)$$

where  $\Delta\hat{\theta} = \hat{\theta}(t) - \hat{\theta}(t-T)$  is the steady-state phase difference between the injection signal phase and the oscillator output phase, and  $\delta$  is the small fluctuation of the phase difference. The upper sign is for parallel-resonant oscillators and the lower sign is for series-resonant oscillators.

The purpose of self-injection locking is to stabilize the oscillator frequency and reduce the phase fluctuation. One may assume that the self-injection-locked oscillator has very small output frequency change and

$$\frac{d\hat{\theta}(t-T)}{dt} - \omega_0 \approx 0. \quad (8)$$

The stability equation (7) for the oscillators becomes

$$\frac{d\delta}{dt} = \mp \rho\omega_{3dB} (\cos \Delta\hat{\theta}) \cdot \delta \quad (9)$$

where the upper sign is for parallel oscillators and the lower sign is for series oscillators.

The phase stability of the self-injection-locked oscillator is determined by the phase delay  $\Delta\hat{\theta}$  in (9). For parallel oscillators, if  $\cos \Delta\hat{\theta} > 0$  ( $-\pi/2 < \Delta\hat{\theta} < \pi/2$ ), the fluctuation of the phase difference between the oscillator output and self-injection signal approaches zero, and the phase difference is stable. If  $\cos \Delta\hat{\theta} < 0$  ( $\pi/2 < \Delta\hat{\theta} < 3\pi/2$ ), the fluctuation of the phase difference between the oscillator output and self-injection signal increases with time. The self-injection locking cannot stabilize the oscillator phase or frequency itself under this condition.

For series oscillators, if  $\cos \Delta\hat{\theta} < 0$  ( $\pi/2 < \Delta\hat{\theta} < 3\pi/2$ ), the fluctuation of the phase difference between the oscillator output and self-injection signal approaches zero, and the phase difference is stable. In real self-injection-locked oscillators, the locking range  $\rho\omega_{3dB} \ll \omega_0$ , the steady-state phase can be approximated by  $\theta(t) \approx \omega_0 t + \theta_0$ . If  $\cos \Delta\hat{\theta} > 0$  ( $-\pi/2 < \Delta\hat{\theta} < \pi/2$ ), the fluctuation of the phase difference between the oscillator output and self-injection signal increases with time, and the phase difference is unstable. The self-injection locking cannot stabilize the oscillator phase or frequency itself under this condition.

The above stability analysis results are summarized in Fig. 3. The stability of the self-injection-locked oscillators depends on the feedback loop delay and the types of the oscillators, which are represented by an equivalent parallel- or series-resonant circuit model. The results are similar to the mode stability analysis of parallel-coupled phase-locked oscillator arrays [12], where the stable steady-state phase differences between the oscillator elements depend on the coupling phase and oscillator circuit models.

#### IV. EXPERIMENTAL RESULTS

An oscillator was used for the experimental verification of this paper's theory. The oscillator is a varactor-tuned MESFET

		Oscillator Model	
		Parallel	Series
Self Injection Locking Stability	Stable	$-90^\circ < \Delta\hat{\theta} < 90^\circ$	$90^\circ < \Delta\hat{\theta} < 270^\circ$
	Unstable	$90^\circ < \Delta\hat{\theta} < 270^\circ$	$-90^\circ < \Delta\hat{\theta} < 90^\circ$

Fig. 3. Stability conditions between the equivalent oscillator model and loop phase. For the parallel-resonant oscillators, the stable output phase requires  $-90^\circ < \Delta\hat{\theta} < 90^\circ$  (i.e.,  $\cos \Delta\hat{\theta} > 0$ ). For series-resonant oscillators, the stable output phase requires  $90^\circ < \Delta\hat{\theta} < 270^\circ$  (i.e.,  $\cos \Delta\hat{\theta} < 0$ ).

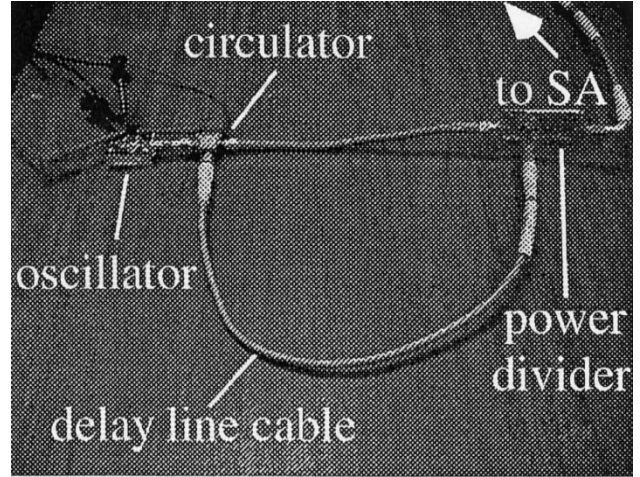


Fig. 4. Experimental setup of the self-injection-locked oscillator with a delay-line cable in the feedback loop. The attenuator can be inserted into the feedback loop to change the self-injection signal strength.

voltage-controlled oscillator (VCO) with a nominal tuning range of 8–9 GHz. The VCO uses an NE32184A packaged MESFET and M/A-COM 46600 varactor diode, and is fabricated on a Rogers Duroid board 5880 ( $\epsilon_r = 2.2$ ) with a thickness of 0.787 mm. The output power of the oscillator is  $P_0 = 5.5$  dBm. The  $Q$  factor of the oscillator can be decided from the injection-locking range [5], [6], and  $Q = 17$  at  $\omega_0 = 8.5$  GHz. The oscillator is the same design as the parallel-resonant oscillator in coupled oscillator arrays with the coupling phase  $\Phi = 0$  or  $2\pi$ . The oscillator is parallel resonant, which is verified by the coupled oscillator array with an antenna array by testing its radiation patterns [12].

The measurement setup shown in Fig. 4 is similar to Fig. 1, and one output port of the power divider is connected to the Agilent Spectrum Analyzer E4407B with phase-noise measurement personality (option 226) [13]. Agilent E4407B can make a log plot of phase-noise measurement in ten successive spectrum sweeps. The stability of the oscillator output phase is related to the phase noise. From the previous derivations, we know the phase noise is indeed the ensemble average of power spectral density of the phase fluctuations. If the oscillator is very stable, its phase noise will be very low and the phase fluctuation is also very small [5], [6]. If the measured signal has enough residual or unstable FM that the resolution bandwidth misses the peak on some of the sweep, the measured power will be incorrect and the trace will have discontinuities or an abrupt change of the

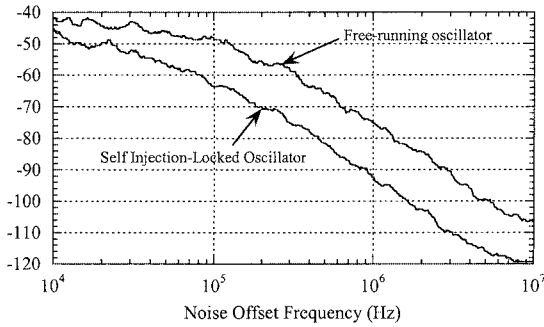


Fig. 5. Measured phase noise of the free-running parallel-resonant oscillator and the same oscillator with a self-injection-locked signal (total loop delay of 15.7 ns and self-injection signal 7 dBm lower than the oscillator free-running output power  $P_0 = 5.5$  dBm). The free-running frequency of the oscillator is 8.193 GHz, and is shifted from 8.193 to 8.150 GHz after applying the self-injection signal. The frequency shift is due to the change of the oscillator output load. The phase noise is reduced after self-injection locking, as compared to the phase-noise trace of the free-running oscillator.

slope [13]. Here, we will use this property to test the stability of self-injection-locked oscillators for different feedback loop delays. We also use time-domain reflectometer (TDR) to test the delays of the individual components in the loop, including the cable connecting the oscillator and circulator, delay within circulator ports, connection between the circulator and power divider, delay within power-divider ports, and feedback cable between the power divider and circulator, and then add up those delays in the feedback loop for the total delay. We can put different attenuators in the feedback loop to change the self-injection signal strength, and observe the self-injection-locked oscillator output spectrum while keeping the whole loop to satisfy the stability conditions.

First, we want to test the phase noise of the free-running oscillator without any self-injection signal. One output port of the power divider is connected to the spectrum analyzer, and the other output port is terminated with a 50- $\Omega$  load. The self-injection signal port of the circulator is also terminated with a 50- $\Omega$  load. The free-running frequency of the oscillator is 8.193 GHz, and the measured phase-noise result is shown in Fig. 5. The curve shown in this figure is the smoothed one. In typical phase-noise measurements, there are often some spikes on the phase-noise traces caused by the power-supply instability or the external interferences. The noise or spike reduction on the phase-noise traces can be accomplished by using the smoothing or averaging or filtering function in the phase-noise measurement personality (option 226) [13].

When we apply the self-injection signal on the oscillator, there is a slight frequency shift from 8.193 to 8.150 GHz due to the change of oscillator output load. Here, we select two cables of different length in the feedback loop, and have total delay time of 4.0 and 15.7 ns with the total insertion loss of 7.0 dBm at 8.150 GHz. For the total loop delay 15.70 ns,  $\cos \Delta\hat{\theta}_p = 0.9603$  at the frequency of 8.150 GHz, and the phase-noise result is shown in Fig. 5. We find the output spectrum is stable for the loop delay of 15.7 ns (i.e.,  $\cos \Delta\hat{\theta}_p > 0$ ) and the phase noise is less than the free-running one (Fig. 3) [11]. This confirms our stability analysis.

For the total loop delay of 4.0 ns (i.e.,  $\cos \Delta\hat{\theta}_p < 0$ ), the phase-noise results measured at different times are shown in

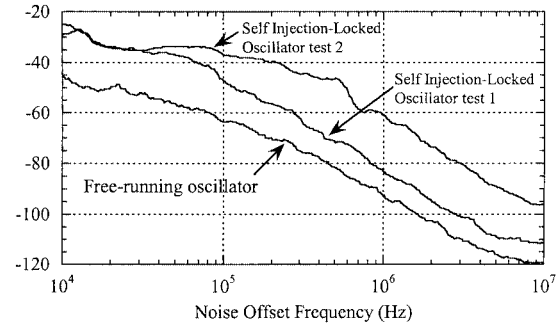


Fig. 6. Measured phase noise of the free-running parallel-resonant oscillator and the same oscillator with a self-injection-locked signal (total loop delay of 4.0 ns and self-injection signal 7 dBm lower than the oscillator free-running output power  $P_0 = 5.5$  dBm) tested at different times. The oscillator frequency is shifted from 8.193 to 8.150 GHz after applying the self-injection signal. The frequency shift is due to the change of the oscillator output load. The output phase noise is noisy after self-injection locking, as compared to the phase-noise trace of the free-running oscillator. There is some discontinuity or abrupt change of the slope in the trace due to the residual or unstable FM in the signal. The result shows the oscillator output phase is unstable under such self-injection condition.

Fig. 6. The phase noise of the self-injection-locked oscillator is unstable and noisy. The phase-noise results also have discontinuities or abrupt change of the slope in the traces. For all the phase-noise measurements here, we keep the same output power from the self-injection-locked oscillator setup to the spectrum analyzer. This again confirms our oscillator phase stability analysis of self-injection-locked oscillators.

## V. CONCLUSIONS

Stability analysis in a self-injection-locked oscillator has been derived. The stability analysis is developed for different oscillator models and the feedback loop phase. The stability analysis is similar to that of coupled oscillator arrays, and we use the perturbation analysis around the steady states of the phase dynamics. The output parallel-resonant oscillator phase is stable for total loop phase  $\cos \Delta\hat{\theta}_p > 0$ , and the series-resonant oscillator is stable for total loop phase  $\cos \Delta\hat{\theta}_s < 0$ . The self-injection locking can also be used to test the oscillator circuit model when the self-coupling phase is known.

The theory is verified by using a self-injection-locked GaAs MESFET oscillator operating at the X-band with delay loops. We use the phase-noise properties to test the stable self-injection-locked oscillator. If the measured signal has enough residual FM that the resolution bandwidth misses the peak on some of the sweep, the measured power will be incorrect and the trace will have discontinuities or an abrupt change of the slope [13]. We use two different loop phases (two cables of different length) and measure the phase noise. We find the phase-noise result of the parallel-resonant oscillator is stable for the loop with  $\cos \Delta\hat{\theta}_p > 0$ , but unstable and noisy and has discontinuities or an abrupt change of the slope on the traces with  $\cos \Delta\hat{\theta}_p < 0$ . Those confirm our stability analysis.

There are several aspects of the stability analysis not treated in this paper. The first is the influence of amplitude dynamics and AM-to-PM conversion. The amplitude of the oscillator is assumed constant and those effects are neglected in this paper. In the derivation of the phase perturbation around the steady states

in self-injection-locked oscillators, we use the assumption that the loop phase is constant with respect to time. However, there is still some small variation of the oscillator frequency due to the oscillator output load change or intrinsic oscillator phase-noise sources. A part of the oscillator output signal is passed through the feedback loop and fed back to the oscillator itself, and tries to stabilize the frequency and phase dynamically for certain feedback delay ranges. If the loop phase is not constant with respect to time, the assumption of the constant loop phase for the self-injection-locked oscillator will not be valid. We should study the effects of the oscillator frequency shift and loop phase variation carefully in the future.

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